

Introduction to the Axioms of Topology

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Hi all! Welcome to the 4th installment of Matt's Math Mondays! Today, we will introduce the basic ideas behind topology.

Consider the collection of all people living on Earth. Such a set would contain you, me, or some dude in Texas named Billy. Saying someone lives on Earth is a valid statement but it doesn't exactly tell us anything. I can then consider the collection of all people that live in the United States. Saying someone lives in the United States indeed gives me more information.

Observe that United States is a subset of the Earth, more precisely, for any person that lives in the United States, that person also lives on Earth. This provides some useful motivation for talking about space in an abstract but useful way. Let X be any set. We may consider an $y \in X$ as equivalent to saying y "lives" in X . The X in this case takes on the place of the Earth. So now we talk about subsets A of X . Moreover we would like to talk about a collection of subsets of X . We declare a new set $T = \{U \mid A \subset X\}$ (We read $A \subset X$ as U is a subset of X .) We may declare T as a topology on X and the ordered pair (X, T) as a topological space of X , if (X, T) obey the following axioms.

1. $\emptyset, X \in T$. We read this as both the set (\emptyset) that contains nothing and the whole set themselves (X) are elements of T . In our example, this is equivalent to saying the we may discuss people being nowhere and also people living on Earth.
2. Consider the some family of subsets of X (which may be infinite or finite) $\alpha = \{A_1, A_2, \dots\}$, each of which are elements of T . It follows that their union $\cup \alpha$ is also an element of T . Union means or. In our example, if we may talk about someone living in the United States and someone living in Egypt, we may talk about someone living in the United States or in Egypt.
3. Consider a FINITE family of subsets of X , $\beta = \{B_1, B_2, \dots B_n\}$, then their intersection $\cap \beta$ is also an element of T . In our example, if we may talk about someone living in California and living in the United States, we may then discuss someone living both in California and the United States.