Some Quick Logic Proofs

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Hello everybody! This week I have finals so I have decided to just extend my one of my previous posts by doing some proofs to show you how to write a proof.

1. $\forall y Rby \models \exists x Rxx$ Suppose that there is an interpretation I such that

(1.1) $\forall y Rby$ is true in I.
(1.2) $\exists x Rxx$ is false in I.
We will deal with (1.2) first. For (1.2) to hold, we require that
(1.2.1) $\forall x (x, x) \notin R$
For (1.1) to hold, we need
(1.1.1) $\forall y, (b, y) \in R$

But if we take, $y = b$, we observe that (1.1.1) and (1.2.1) are in contradiction. Therefore, $\forall y Rby \models \exists x Rxx$ is a valid logical consequence.

2. $\forall x((Rxx \leftrightarrow Raa) \wedge Rax \models \forall x Rxx$ Suppose there is an interpretation I such that

(2.1) $\forall x((Rxx \leftrightarrow Raa) \wedge Rax$ is true in I
(2.2) $\forall x Rxx$ is false in I.
For (2.2) to hold, we need
(2.2.1) $\exists x, (x, x) \notin R$
For (2.1) to hold we need one of the following
(2.1.1) $\forall x(Rxx \leftrightarrow Raa)$
or
(2.1.2) $\forall x, Rax$
By (2.1.2), we have
(2.1.2.1) $\forall x, (a, x) \in R$
If we fix $a \neq x$, we guarantee that (2.1.2.1) and (2.2.1) will not contradict.
Then we can construct a countermodel as such,
$D = \{1, 2\}$
$\text{ext}R = \{(1, 1), (1, 2)\}$
$a = \{1\}$
$b = \{2\}$